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## **Title** : Uniform open-image theorem - applications

**Abstract**: (joint work with Akio Tamagawa - RIMS) Let k be a field of finite type over  $\mathbb{Q}$  and S a curve (smooth, separated and geometrically connected) over k. We are interested in  $\ell$ -adic representations  $\rho: \pi_1(S) \to GL_n(\mathbb{Z}_\ell)$  of the étale fundamental group of S. Let us write G for the image of  $\rho$ ,  $\mathcal{G}$  for the image of the geometric étale fundamental group of S and, for a closed point s of S,  $G_s$  for the image (well-defined up to conjugacy) of a decomposition group at s. We show that for any positive integer d the set of all closed point on S with residue degree  $\leq d$ and where  $G_s$  has codimension > 2 in G is finite. If we furthermore assume that the abalianization of any open subgroup of  $\mathcal{G}$  is finite, we show that for any positive integer d the set  $E_d$  of all closed point on S with residue degree  $\leq d$  and where  $G_s$  is not open in G is finite and that the index of  $G_s$  in G is uniformly bounded outside  $E_d$  (uniform open image theorem). We will give a few applications of this result, especially to the problem of bounding uniformly the  $\ell$ -primary torsion of higher dimensional abelian varieties and to Andr's theorem about the variation of motivated motivic Galois group. If time allows, we will also sketch the main steps of the proof of the uniform open image theorem.