ABELIAN VARIETIES & GALOIS ACTIONS

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Faculty of Mathematics and Computer Science
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ABSTRACTS

Speaker : Yuri Zarhin
Title : *Homomorphisms of abelian varieties over cyclotomic extensions*
Abstract : We discuss homomorphisms of abelian varieties over infinite cyclotomic extensions of finitely generated fields.

Speaker : Marc Hindry
Title : *On the number of torsion points of abelian varieties over number fields*
Abstract: I will discuss joint work with Nicolas Ratazzi on how the number of torsion points of a fixed abelian variety A defined over a number field K may increase when the degree of K tends to infinity. The answer (proved or conditional) depends on the Galois representations attached to A and its Mumford-Tate group. I will discuss both known cases and what we expect in general.

Speaker : Chris Hall
Title : *Expander finiteness and expanders in arithmetic geometry*
Abstract : Let K be the rational function field $K = \mathbb{Q}(t)$ and let A/K be an abelian variety. We may regard A as a one-parameter family of abelian varieties (over $\mathbb{Q}$), and it is natural to ask how the monodromy groups of the corresponding Tate modules vary in the family. In this talk we will sketch diophantine techniques which give a handle on the problem. We will then sketch how to apply the techniques to show that if A is (geometrically) simple and non-isotrivial over K, then there are only finitely many parameters defined over a number field of bounded degree for which the corresponding fiber is not simple. Faltings' theorems play a key role, but as we will explain, recent advances in expander graphs play a significant role as well.

Speaker : Sara Arias-de-Reyna
Title : *On a conjecture of Geyer-Jarden about abelian varieties over finitely generated fields*
Abstract : Let A be an abelian variety defined over a finitely generated field K. Around 1978 W-D. Geyer and M. Jarden proposed a conjecture about the torsion points of A that are defined over certain infinite algebraic extensions of K. In this talk we will show that this conjecture holds when A has big monodromy. This is a joint work with W. Gajda and S. Petersen.

Speaker : Anna Cadoret
Title : *Uniform open-image theorem - applications*
Abstract: (joint work with Akio Tamagawa - RIMS) Let k be a field of finite type over $\mathbb{Q}$ and S a curve (smooth, separated and geometrically connected) over k. We are interested in ℓ-adic representations $\rho : \pi_1(S) \to GL_n(\mathbb{Z}_\ell)$ of the étale fundamental group of S. Let us write G for the image of ρ, $\mathcal{G}$ for the image of the geometric étale fundamental group of S and, for a closed point s of S, G_s for the image (well-defined up to conjugacy) of a decomposition group at s. We show that for any positive integer d the set of all closed point on S with residue degree $\leq d$ and where G_s has codimension > 2 in G is finite. If we furthermore assume that the abalianization of any open subgroup of $\mathcal{G}$ is finite, we show that for any positive integer d the set E_d of all closed point on S with residue degree $\leq d$ and where G_s is not open in G is finite and that the index of G_s in G is uniformly bounded outside E_d (uniform open image theorem). We will give a few applications of this result, especially to the problem of bounding uniformly the ℓ-primary torsion of higher dimensional abelian varieties and to Andr's theorem about the variation of motivated motivic Galois group. If time allows, we will also sketch the main steps of the proof of the uniform open image theorem.

Speaker : Moshe Jarden
Title : *Diamonds in torsion of abelian varieties*
Abstract: We consider the following conjecture:

Conjecture: *Let K be a Hilbertian field, A/K an abelian variety, and M a field extension of K in $K(A_{tor})$. Then M is Hilbertian.*

We shall explain the basic principles of the proof of the conjecture when K is finitely generated over its prime field.

Speaker : Arno Fehm
Title : *Domains with Hilbertian quotients fields*
Abstract : Hilbert's irreducibility theorem for number fields is of central importance in Galois theory and arithmetic geometry and led to the notion of a Hilbertian field, that is, a field that satisfies the consequence of Hilbert's theorem. We investigate sufficient conditions on the arithmetic of an integrally closed domain R that

imply that the quotient field of R is Hilbertian, thereby extending a result of Weissauer on generalized Krull domains of dimension exceeding one. As an application we get that certain fields of power series are Hilbertian and deduce that every finite group can be realized as a Galois group over them. (Joint work with Elad Paran.)

Speaker : Otmar Venjakob
Title : *SK_1 of Iwasawa algebras*
Abstract : In non-commutative Iwasawa theory K-theoretic properties of Iwasawa algebras, i.e., completed group algebras of, e.g., p-adic Lie groups G, play a crucial role. Such groups arise naturally as Galois groups attached to p-adic representations as for example on the Tate module of abelian varieties. In this talk we shall discuss over all the vanishing of SK_1-groups of such Iwasawa algebras. An arithmetic application is the partial uniqueness of '(non-commutative) p-adic L-functions.

Speaker : Werner Bley
Title : *The equivariant Tamagawa number conjecture for elliptic curves*
Abstract : Let $E/\mathbb{Q}$ be an elliptic curve and $K/\mathbb{Q}$ a finite Galois extension with group G. The ETNC for the pair $(h_1(E/K)(1); \mathbb{Z}[G])$ is an equivariant refinement of the Birch and Swinnerton-Dyer conjecture. We will try to give an explicit formulation of the ETNC in this context and explain how this can be used to obtain both numerical and, in very special cases, also theoretical evidence for the validity of the conjecture.

Speaker : Lior Bary-Soroker
Title : *On p-embedding problems in characteristic p*
Abstract : Studying absolute Galois groups via embedding problems started with Iwasawa. Today there is a lot of activity on this subject. The main objective of this talk is to present a recent result of the speaker and Duy-Tan. We prove that an embedding problem with p-kernel for a field of characteristic p is solvable under the mild condition that the field admits a valuation which is not p-divisible. All notions will be explained in the talk.

Speaker : Francesc Bars
Title : *On cyclotomic Iwasawa theory for function fields*
Abstract : Let F be a global function field of characteristic p (for simplicity in the talk we will restrict to $F = \mathbb{F}_q(T)$). Let us consider $\mathcal{F}$ the "cyclotomic" extension given by the $\mathfrak{p}^n$-torsion of the Carlitz module, $\mathfrak{p}$ a prime ideal of F, we recall that $Gal(\mathcal{F}/F) \cong \mathbb{Z}_p^{\mathbb{N}} \times A$ where A is a finite group of order prime to p.

First, we will consider this "cyclotomic" extension over the local field $F_{\mathfrak{p}}$. Coleman power series are a useful tool to deal with norm compatible systems of units. In this "cyclotomic" setting they provide us different results: an analog of Wiles's

reciprocity law and an efficient way to define the Coates-Wiles homomorphism; also, we can compute the image of this homomorphism for particular norm compatible systems of "cyclotomic" units.

Secondly, we will present various aspects of Iwasawa theory for characteristic ideals and the Pontrjagin dual of the Selmer group of an abelian variety A defined over F associated with a $\mathbb{Z}_p^{\mathbb{N}}$-extension, providing algebraic ingredients for the Main Conjecture.

Speaker : Piotr Krasoń
Title : *On arithmetic in Mordell-Weil groups*
Abstract : (joint with G.Banaszak) We will describe the problem of detecting linear dependence of points in Mordell-Weil groups $A(F)$ of abelian varieties. This is done via reduction maps. We determine the sufficient conditions for the reduction maps to detect linear dependence in $A(F)$. We also show that our conditons are very close to be or perhaps are the best possible. In particular we try to determine the conditions for detecting linear dependence in Mordell-Weil groups via finite number of reductions. The methods combine applications of transcedental, ℓ-adic and mod v techniques.

Speaker : Cristian Popescu
Title : *An Equivariant Main Conjecture in Iwasawa Theory and Applications*
Abstract : In recent joint work with Greither, we proved an equivariant main conjecture in the Iwasawa theory of arbitrary global fields, under the assumption that the classical Iwasawa mu-invariants vanish. This is a refinement of earlier results of Wiles (in characteristic 0) and Deligne-Tate (in positive characteristic.) In the formulation and proof of our main conjecture, a crucial role is played by the Galois module structure of the l-adic realizations of certain (abstract) 1-motives.

In this talk, we will discuss the statement and sketch the proof of our main conjecture. Also, we will give various applications towards (refinements of) the Brumer-Stark and Coates-Sinnott conjectures, explicit l-adic models for Tate sequences and the Equivariant Tamagawa Number conjecture for Dirichlet motives.